5.6 Radical Expressions

Properties

$a, b, \sqrt[n]{a}, \sqrt[n]{b} \in \mathbb{R}$

$m, n \in Z$

1. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
3. $\sqrt[n]{b^m} = (\sqrt[n]{b})^m$
4. $\sqrt[mn]{b} = \sqrt[m]{\sqrt[n]{b}} = \sqrt[n]{\sqrt[m]{b}}$

Ex:

$\sqrt{36} = \sqrt{9} \cdot \sqrt{4} = 3 \cdot 2 = 6$

$\sqrt{4^3} = \sqrt{4}^3 = 2^3 = 8$

$\sqrt[8]{16}$ $\sqrt[4]{\sqrt{16}} = \sqrt[4]{4} = \sqrt{\sqrt{4}} = \sqrt{2}$

Ex:

$\sqrt[3]{8^2} = \sqrt[3]{8}^2 = 2^2 = 4$

$\sqrt[3]{216} = \sqrt[3]{8} \cdot \sqrt[3]{27} = 6$

$\sqrt[6]{64} = \sqrt[3]{\sqrt{64}} = \sqrt[3]{8} = 2$

Ex:

$\sqrt[15]{32}$ $\sqrt[3]{\sqrt[5]{2^5}} = \sqrt[3]{\sqrt[5]{32}} = \sqrt[3]{2}$

$2^5 = 32$

$2 \quad 16$

$4 \quad 4$

$2 \quad 2$

$\sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{\sqrt{4}} = \frac{\sqrt{7}}{2}$

Rationalize the Denominator
(free of irrational numbers)

$\sqrt{\frac{7}{3}} = \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{21}}{3}$

$\frac{9}{\sqrt{b^3}} = \frac{9}{b\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{9\sqrt{b}}{b^2}$

$\frac{4}{\sqrt[4]{27a^2}} = \frac{4}{\sqrt[4]{3^3a^2}} \cdot \frac{\sqrt[4]{3^1a^2}}{\sqrt[4]{3^1a^2}} = \frac{4\sqrt[4]{3a^2}}{\sqrt[4]{3^4a^4}} = \frac{4\sqrt[4]{3a^2}}{3a}$

$\frac{2}{\sqrt[5]{2c^4}} \cdot \frac{\sqrt[5]{2^4c^1}}{\sqrt[5]{2^4c^1}} = \frac{2\sqrt[5]{2^4c}}{2c}$

$\sqrt[5]{2^5c^5} = \frac{\sqrt[5]{2^4c}}{c}$

$\frac{1}{\sqrt[5]{a^2b^3c}} \cdot \frac{\sqrt[5]{a^3b^2c^4}}{\sqrt[5]{a^3b^2c^4}} = \frac{\sqrt[5]{a^3b^2c^4}}{abc}$

Operations (combine like terms)

ex:

$\sqrt{50} - 3\sqrt{72} + \sqrt[3]{8}$

$5\sqrt{2} - 18\sqrt{2} + 2$

$-13\sqrt{2} + 2$

$2 - 13\sqrt{2}$

ex:

$\sqrt{8} + \sqrt{98}$

$2\sqrt{2} + 7\sqrt{2}$

$= 9\sqrt{2}$

ex:

$\sqrt[3]{81} - \sqrt[3]{24}$

81 → 27 · 3 → 9 · 3 → 3 · 3 · 3

$3\sqrt[3]{3} - 2\sqrt[3]{3}$

$\sqrt[3]{3}$

ex:

$(\sqrt{3} - \sqrt{5})^2$

$(\sqrt{3} - \sqrt{5})(\sqrt{3} - \sqrt{5})$

$-\sqrt{15} \quad -\sqrt{15}$

$3 - 2\sqrt{15} + 5$

$8 - 2\sqrt{15}$

ex:

$(\sqrt{7} - \sqrt{11})(\sqrt{7} + \sqrt{11})$

$7 - 11$

-4

ex:

$\frac{1}{3+\sqrt{2}} \cdot \frac{3-\sqrt{2}}{3-\sqrt{2}}$

Conjugate $a+b$ $a-b$

$9 - 2$

$\frac{3-\sqrt{2}}{7}$

ex: Conjugates $a + b$ and $a - b$

$\frac{8}{4-\sqrt{5}} \cdot \frac{4+\sqrt{5}}{4+\sqrt{5}}$

ex:

$2\sqrt{7} \cdot \sqrt{7} = 2\sqrt{49} = 2 \cdot 7 = 14$

$\frac{3+\sqrt{7}}{5-2\sqrt{7}} \cdot \frac{5+2\sqrt{7}}{5+2\sqrt{7}} = \frac{15+6\sqrt{7}+5\sqrt{7}+14}{-3}$

$25 - 4 \cdot 7$

-3

$= \frac{29+11\sqrt{7}}{-3}$

$= \frac{-29-11\sqrt{7}}{3}$

Fun with Factoring

ex:

$x^2 - 18$

$(x)^2 - (\sqrt{18})^2$

$3\sqrt{2}$

$(x + 3\sqrt{2})(x - 3\sqrt{2})$

HW

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